

## Hardy - Weinberg Problems

Recall that the gene pool describes ALL the available genes (meaning all the versions of each gene- the alleles) in a population. Two scientists named G.H. Hardy and Wilhelm Weinberg found a way to mathematically describe gene pools and show change over time by showing change in allelic frequencies. If we know the frequency of the phenotypes, we can work backwards to find the frequency of the alleles.

If we were to study a SINGLE gene trait with two alleles, we can describe the frequencies of the alleles in the gene pool by using the formula:

$$p + q = 1$$

p represents the frequency of the dominant allele, and q represents the frequency of the recessive allele. These frequencies should stay constant in a non-evolving population. The alleles can combine in any pattern (p with p, p with q or q with q), but the frequency will stay constant.

We cannot outwardly observe alleles, but we can observe the phenotypes displayed by the expression of the alleles. Every organism has two alleles for any particular gene. The chance of a gamete with a dominant allele will meet with another gamete with a dominant allele is  $p \times p$ , or  $p^2$  (frequency of homozygous dominant individuals). The chance of a gamete with a recessive allele will meet with another gamete with a recessive allele is  $q \times q$ , or  $q^2$  (frequency of recessive individuals). The chance of a dominant sperm meeting with a recessive egg is  $p \times q$ , or  $pq$ . The chance of a dominant egg being fertilized with a recessive sperm is  $q \times p$ , or  $pq$ . Both of those situations will result in a heterozygous individual, so we can combine them into  $2pq$  (frequency of heterozygous dominant individuals). All of the frequency of individuals in the population can be expressed in the formula

$$p^2 + 2pq + q^2 = 1$$

If given an allele frequency, we can find the individual frequencies, and if given individual frequencies, we can find the allele frequencies. Keep in mind that  $p^2$  and  $2pq$  both display the dominant phenotype. It is always easiest to start with the recessive allele/individuals ( $q/q^2$ ).

What does p stand for? \_\_\_\_\_

What does q stand for? \_\_\_\_\_

What is  $p^2$ ? \_\_\_\_\_

What is  $2pq$ ? \_\_\_\_\_

What is  $q^2$ ? \_\_\_\_\_

Identify ALL 5 components for each. Solve for the parts you do not know. Show all of your work.

1. In a population of timber rattle snakes, there are two coloration varieties yellow, which is dominant, and black. The frequency of the yellow allele is 0.9 in the

population. Please identify the following:

p=  
q=  
p<sup>2</sup>=  
2pq=  
q<sup>2</sup>=

So... what is the frequency of the yellow phenotype? \_\_\_\_\_

2. Polydactyly (extra fingers or toes) in humans is a dominant trait. Roughly 1% of the population is born with extra digits. Please identify the following:

p=  
q=  
p<sup>2</sup>=  
2pq=  
q<sup>2</sup>=

So... what is the frequency of the polydactyly gene in the population? \_\_\_\_\_

3. If 9% of an African population is born with a severe form of sickle cell anemia (ss), (Identify what you know, solve for what you don't know)

p=  
q=  
p<sup>2</sup>=  
2pq=  
q<sup>2</sup>=

So... what percentage of the population will be more resistant to malaria because they are heterozygous (Ss) for sickle-cell gene? \_\_\_\_\_-

4. In a population of 500 mice, there are 180 that display the recessive phenotype for the tan pelage. The brown pelage is coded for by a dominant trait. Please tell me the following:

p=  
q=  
frequency of heterozygous mice=  
frequency of homozygous dominant mice=  
freq. of recessive mice=

5. This is a classic data set on wing coloration in the scarlet tiger moth (*Panaxia dominula*). Coloration in this species had been previously shown to behave as a single-locus (one gene), two-allele system with incomplete dominance. Data for 1612

individuals are given below. This population is NOT in equilibrium. Find p and q FIRST (you can count the alleles!), and then solve for the  $p^2$ ,  $2pq$ , and  $q^2$ .

White-spotted (AA) = 1469 Intermediate (Aa) = 138 Little spotting (aa) = 5

p=  
q=  
 $p^2$ =  
 $2pq$ =  
 $q^2$ =

Assuming the population reaches equilibrium in the next generation, what percent of the population will have the intermediate coloration? \_\_\_\_\_

6. In a population of peppered moths, there were 340 light moths found, and 27 dark moths found. Assume that the allele for the dark color is recessive. Define all 5 components of the Hardy-Weinberg populations.

p=  
q=  
 $p^2$ =  
 $2pq$ =  
 $q^2$ =

7. A very large population of randomly-mating laboratory mice contains 35% white mice. White coloring is caused by the double recessive genotype, "aa". The dominant phenotype is a grayish brown. Calculate allelic and genotypic frequencies for this population.

p=  
q=  
 $p^2$ =  
 $2pq$ =  
 $q^2$ =

Populations in Hardy Weinberg equilibrium are NOT evolving- that is why we can assign mathematical values to the allelic frequencies. Suppose one phenotype was at an extreme disadvantage and could not reproduce...

We can assign a value to each phenotype's fitness. If they all had an equal chance of survival, their fitness level would be 1; if there is no chance of survival, the fitness level will be 0. Fitness is multiplied to the individual, not the allele.

Imagine a population where  $p^2$  and  $2pq$  individuals had a fitness level of 1, and  $q^2$  had a fitness of 0. The starting allele frequencies are  $p=0.8$  and  $q=0.2$ . That means the dominant individuals ( $p^2 + 2pq$ ) is 0.96 and  $q^2 = 0.04$ . Multiply the number of individuals by their fitness level and recalculate the allele frequencies. If you only have frequencies, imagine you had 1000 individuals- it is easy to calculate that way. Recalculate and now  $p = 0.816$  and  $q = 0.1835$ . Has evolution occurred? The allele frequency changed- so YES!

8. Achondroplasia is a dominant trait. The homozygous dominant phenotype is lethal (Fitness level of 0). The normal height has a fitness level of 1 and the heterozygous phenotype has the fitness level of 0.5. Achondroplasia has a frequency of 1 in 15,000 births, but in this one isolated village, its frequency was 1 in 200. What is the frequency of the  $p$  allele and  $q$  allele? Show the change over two generations.

9. Cystic fibrosis is a recessive trait. Suppose the fitness level of CF is 0.2. Show the change in allelic frequencies over 3 generations if you know that currently 1% of the population has CF. What do you notice about the allele frequency?